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QCD Corrections to the Nucleon's Spin Structure Function $g_2(x, Q^2)$

JIRO KODAIRA AND YOSHIKI YASUI*

*Dept. of Physics, Hiroshima University
Higashi-Hiroshima 739, JAPAN*

KAZUHIRO TANAKA

*Dept. of Physics, Juntendo University
Inba-gun, Chiba 270-16, JAPAN*

TSUNEO UEMATSU†

*Dept. of Fundamental Sciences
FIHS, Kyoto University
Kyoto 606-01, JAPAN*

Abstract

We investigate the renormalization of the twist-3 operators which are relevant for the spin-dependent structure function $g_2(x, Q^2)$. We derive the anomalous dimension for the non-singlet part by calculating the off-shell Green's functions of the twist-3 operators including the operators which are proportional to the equation of motion.

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The nucleon spin structure observed in the deep inelastic scattering is described by the two structure functions $g_1(x, Q^2)$ and $g_2(x, Q^2)$. In the framework of the operator product expansion and the renormalization group, not only the twist-2 operators but also the twist-3 operators contribute to g_2 in the leading order of $1/Q^2$ [1]. The general feature characteristic to the higher-twist operators is the occurrence of the operators which are proportional to the equation of motion (EOM operators) [2]. Analyzing the twist-3 operators must be interesting and useful from the theoretical viewpoint, since they are the simplest examples of the higher-twist operators.

There have been a lot of works [3, 4, 5] on the Q^2 -evolution of g_2 . We here mention the two works which are frequently cited. (i) Ji and Chou [3] computed anomalous dimensions of the twist-3 operators for g_2 in the Feynman gauge (for general spin n). They employed the massless on-shell scheme to compute the three-point function. However, it is not clear how the EOM operators are dealt with since their scheme itself might not be consistent due to the infrared singularities coming from the collinear configurations. (ii) Bukhvostov, Kuraev and Lipatov[4] derived GLAP-type evolution equation. But, this was carried out in the axial gauge and the relation to the covariant approach is unclear.

The results of these two works seem not to be identical. Thus, the computation of the anomalous dimensions in a covariant gauge in a fully consistent scheme is desirable. To do this, we decided to compute the off-shell Green's functions to renormalize the operators. In this case, the EOM operators should be included as independent operators. Infrared cut-off is provided by the external off-shell momenta. Recently this scheme has been employed in Ref.[6] and the complete calculation of the anomalous dimensions for the lowest ($n = 3$) moment was demonstrated. The consistency and the efficiency of the method were also argued. Here we extend the computation to the case for the general moment n . We consider the flavor non-singlet case.

Phenomenologically, the first data of g_2 via the polarized deep inelastic scattering have been reported recently [7]. The extensive study will be performed in HERMES. The theoretical determination of the Q^2 -dependence of g_2 is indispensable to extract the physical information from experimental data. Also, the Q^2 -dependence itself will be checked and provide a deeper test of QCD. In view of these, it is extremely important to establish the theoretical prediction, which is yet controversial as discussed above.

We follow the convention of ref.[6]. Let us first list up the gauge invariant twist-3 operators which contribute to the moment $\int_0^1 dx x^{n-1} g_2(x, Q^2)$. In the following expressions, we suppress the flavor matrices λ_i for the quark field ψ .

$$R_{n,F}^{\sigma\mu_1\cdots\mu_{n-1}} = \frac{i^{n-1}}{n} \left[(n-1) \bar{\psi} \gamma_5 \gamma^\sigma D^{\{\mu_1} \cdots D^{\mu_{n-1}\}} \psi - \sum_{l=1}^{n-1} \bar{\psi} \gamma_5 \gamma^{\mu_l} D^{\{\sigma} D^{\mu_1} \cdots D^{\mu_{l-1}} D^{\mu_{l+1}} \cdots D^{\mu_{n-1}\}} \psi \right] - \text{traces} , \quad (1)$$

$$R_{n,l}^{\sigma\mu_1\cdots\mu_{n-1}} = \frac{1}{2n} (V_l - V_{n-1-l} + U_l + U_{n-1-l}) \quad (l = 1, \cdots, n-2) , \quad (2)$$

$$R_{n,m}^{\sigma\mu_1\cdots\mu_{n-1}} = i^{n-2} \mathcal{S} m_\psi \bar{\psi} \gamma_5 \gamma^\sigma D^{\mu_1} \cdots D^{\mu_{n-2}} \gamma^{\mu_{n-1}} \psi - \text{traces} , \quad (3)$$

$$R_{n,E}^{\sigma\mu_1\cdots\mu_{n-1}} = i^{n-2} \frac{n-1}{2n} \mathcal{S} \left[\bar{\psi} (i \not{D} - m_\psi) \gamma_5 \gamma^\sigma D^{\mu_1} \cdots D^{\mu_{n-2}} \gamma^{\mu_{n-1}} \psi + \bar{\psi} \gamma_5 \gamma^\sigma D^{\mu_1} \cdots D^{\mu_{n-2}} \gamma^{\mu_{n-1}} (i \not{D} - m_\psi) \psi \right] - \text{traces} , \quad (4)$$

where

$$V_l = i^n g \mathcal{S} \bar{\psi} \gamma_5 D^{\mu_1} \cdots G^{\sigma\mu_l} \cdots D^{\mu_{n-2}} \gamma^{\mu_{n-1}} \psi - \text{traces},$$

$$U_l = i^{n-3} g \mathcal{S} \bar{\psi} D^{\mu_1} \cdots \tilde{G}^{\sigma\mu_l} \cdots D^{\mu_{n-2}} \gamma^{\mu_{n-1}} \psi - \text{traces}.$$

In the above equations, (- traces) stands for the subtraction of the trace terms to make the operators traceless and D^μ is the covariant derivative. $\{ \}$ means symmetrization over the Lorentz indices, \mathcal{S} the symmetrization over μ_i and g the QCD coupling constant. m_ψ represents the quark mass (matrix). The operators in (2) contain the gluon field strength $G_{\mu\nu}$ and the dual tensor $\tilde{G}_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\alpha\beta} G^{\alpha\beta}$ explicitly. The above

operators are not all independent but satisfy the following equation,

$$R_{n,F}^{\sigma\mu_1\cdots\mu_{n-1}} = \frac{n-1}{n}R_{n,m}^{\sigma\mu_1\cdots\mu_{n-1}} + \sum_{l=1}^{n-2}(n-1-l)R_{n,l}^{\sigma\mu_1\cdots\mu_{n-1}} + R_{n,E}^{\sigma\mu_1\cdots\mu_{n-1}}. \quad (5)$$

Therefore we can exclude one operator among (1)-(4) to form an independent basis. A convenient choice of the independent operators will be (2), (3) and (4). It is to be noticed that, for the n -th moment, n gauge-invariant operators participate in the renormalization.

We multiply the operators by the light-like vector Δ_{μ_i} to symmetrize the Lorentz indices and to eliminate the trace terms. We then embed the operators $\Delta \cdot R_{n,l}^\sigma \equiv \Delta_{\mu_1} \cdots \Delta_{\mu_{n-1}} R_{n,l}^{\sigma\mu_1\cdots\mu_{n-1}}$ into the three-point function as $\langle 0 | T \Delta \cdot R_{n,l}^\sigma(0) A_\mu(x) \psi(y) \bar{\psi}(z) | 0 \rangle$ and compute the one-loop corrections. We employ the Feynman gauge and renormalize the operators in the $\overline{\text{MS}}$ scheme. To perform the renormalization consistently, we keep the quark and gluon external lines off-shell; in this case, the EOM operator mix through the renormalization as a nonzero operator.

One serious problem in the calculation is the mixing of the (many) gauge non-invariant EOM operators. As explained in Ref.[6], these operators are given by replacing some of the covariant derivatives D^{μ_i} by the ordinary derivatives ∂^{μ_i} in (4). This problem can be overcome by introducing the vector Ω_μ , which satisfies $\Delta^\mu \Omega_\mu = 0$ [8], and by contracting the Green's functions $\langle 0 | T \Delta \cdot R_{n,l}^\sigma(0) A_\mu(x) \psi(y) \bar{\psi}(z) | 0 \rangle$ with this vector. This brings the two merits: Firstly, the tree vertices of the gauge invariant and non-invariant EOM operators coincide. Thus, essentially only one EOM operator is now involved in the operator mixing. Secondly, the structure of the vertices for the twist-3 operators are simplified extremely, and the computation becomes more tractable.

After the contraction with Ω_μ , the tree vertices for $\Delta \cdot R_{n,l}^\sigma$ and $\Delta \cdot R_{n,E}^\sigma$ become

respectively,

$$\mathcal{R}_{n,l}^\sigma = \frac{g}{2n} \hat{q} \left((\hat{p} - \hat{q})^{n-2-l} \hat{p}^{l-1} \gamma^\sigma \not{Q} \gamma_5 \not{\Delta} - (\hat{p} - \hat{q})^{l-1} \hat{p}^{n-2-l} \not{Q} \gamma^\sigma \gamma_5 \not{\Delta} \right) t^a, \quad (6)$$

and

$$\mathcal{E}_n^\sigma = -\frac{n-1}{2n} g \left((\hat{p} - \hat{q})^{n-2} \gamma^\sigma \not{Q} \gamma_5 \not{\Delta} + \hat{p}^{n-2} \not{Q} \gamma^\sigma \gamma_5 \not{\Delta} \right) t^a. \quad (7)$$

Here p and q are the momenta of the incoming quark and the outgoing gluon ($\hat{p} \equiv \Delta \cdot p$, $\hat{q} \equiv \Delta \cdot q$), and t^a is the color matrix normalized as $\text{Tr}(t^a t^b) = \frac{1}{2} \delta^{ab}$.

We present the results of the one-loop calculation for the one-particle-irreducible three-point function with the insertion of $\Delta \cdot R_{n,l}^\sigma$ (Fig. 1). We neglect the contribution of the operator proportional to the quark mass, (3), for the moment. In the following expressions, $C_F = (N_c^2 - 1)/2N_c$, $C_G = N_c$, where we assume N_c color, $S_j = \sum_{k=1}^j 1/k$, and $\epsilon = (4 - D)/2$ with D the space-time dimension.

Diagram (A) gives

$$\begin{aligned} \frac{g^2}{16\pi^2\epsilon} (2C_F - C_G) \left\{ \sum_{k=1}^{l-1} \frac{(-1)^{l-k}}{(n-1)} \frac{n-2}{n-2} \frac{C_{k-1}}{C_{l-1}} \mathcal{R}_{n,k}^\sigma + \frac{1}{n-1} \mathcal{R}_{n,l}^\sigma \right. \\ \left. + \sum_{k=l+1}^{n-2} \frac{(-1)^{l-k}}{(n-1)} \frac{n-2}{n-2} \frac{C_k}{C_l} \mathcal{R}_{n,k}^\sigma \right\}. \quad (8) \end{aligned}$$

Diagram (B) + diagram (C) gives

$$\begin{aligned} \frac{g^2}{16\pi^2\epsilon} \left[\sum_{k=1}^{l-1} \left\{ (2C_F - C_G) \frac{(-1)^{l-k}}{(n-l)} \frac{l-1}{n-k-1} \frac{C_{l-k}}{C_{n-l}} - C_G \frac{n-2l-1}{(n-l)(l+1)} \right\} \mathcal{R}_{n,k}^\sigma \right. \\ \left. + \left\{ 2C_F(2 - S_l - S_{n-l-1}) - \frac{C_G}{l+1} \right\} \mathcal{R}_{n,l}^\sigma \right. \\ \left. + (2C_F - C_G) \sum_{k=l+1}^{n-2} \frac{(-1)^{l-k}}{(l+1)} \frac{n-l-2}{k} \frac{C_{k-l}}{C_{l+1}} \mathcal{R}_{n,k}^\sigma + C_G \frac{n-2l-1}{(n-1)(n-l)(l+1)} \mathcal{E}_n^\sigma \right]. \quad (9) \end{aligned}$$

Diagram (D) + diagram (E) gives

$$\begin{aligned} \frac{g^2}{16\pi^2\epsilon} (2C_F - C_G) \left\{ \sum_{k=1}^{l-1} \frac{2(-1)^k}{l(l+1)(l+2)} \frac{l C_k}{C_k} \mathcal{R}_{n,k}^\sigma + \left(\frac{2(-1)^l}{l(l+1)(l+2)} - \frac{(-1)^l}{n-l} \right) \mathcal{R}_{n,l}^\sigma \right. \\ \left. + \sum_{k=l+1}^{n-2} \frac{(-1)^{n-k}}{n-l} \frac{n-l-2}{n-l} \frac{C_{n-k-2}}{C_{n-k-2}} \mathcal{R}_{n,k}^\sigma + \frac{1}{(n-1)(l+1)(l+2)} \mathcal{E}_n^\sigma \right\}. \quad (10) \end{aligned}$$

Diagram (F) + diagram (G) gives

$$\begin{aligned} \frac{g^2}{16\pi^2\epsilon} C_G \left\{ \sum_{k=1}^{l-1} \left(\frac{l+3}{2l(l+1)} + \frac{(l-2)(l-k+1)}{2l(l+1)(l+2)} - \frac{1}{n-l} \right) \mathcal{R}_{n,k}^\sigma \right. \\ + \left(\frac{l+3}{2l(l+1)} + \frac{l-2}{2l(l+1)(l+2)} - \frac{1}{2(n-l)} \right) \mathcal{R}_{n,l}^\sigma \\ \left. - \sum_{k=l+1}^{n-2} \frac{n-k-1}{2(n-l-1)(n-l)} \mathcal{R}_{n,k}^\sigma - \frac{n-2l-2}{(n-1)(n-l)(l+2)} \mathcal{E}_n^\sigma \right\}. \quad (11) \end{aligned}$$

Diagram (H) gives

$$\begin{aligned} \frac{g^2}{16\pi^2\epsilon} C_G \left\{ \sum_{k=1}^{l-1} \left(\frac{1}{l-k} - \frac{1}{l} - \frac{k}{2l(l+1)} \right) \mathcal{R}_{n,k}^\sigma \right. \\ + \left(1 - S_l - S_{n-l-1} - \frac{1}{2(l+1)} - \frac{1}{2(n-l)} \right) \mathcal{R}_{n,l}^\sigma \\ \left. + \sum_{k=l+1}^{n-2} \left(\frac{1}{k-l} - \frac{1}{n-l-1} - \frac{n-k-1}{2(n-l-1)(n-l)} \right) \mathcal{R}_{n,k}^\sigma \right\}. \quad (12) \end{aligned}$$

The contributions from (3) are easily calculated by considering the quark two-point Green's function [3]. The renormalization constants are determined in the $\overline{\text{MS}}$ scheme.

We summarize the final result in the following matrix form:

$$\begin{pmatrix} R_{n,l} \\ R_{n,m} \\ R_{n,E} \end{pmatrix}_B = \begin{pmatrix} Z_{lj} & Z_{lm} & Z_{lE} \\ 0 & Z_{mm} & 0 \\ 0 & 0 & Z_{EE} \end{pmatrix} \begin{pmatrix} R_{n,j} \\ R_{n,m} \\ R_{n,E} \end{pmatrix}_R, \quad (l, j = 1, \dots, n-2), \quad (13)$$

where the suffix $R(B)$ denotes renormalized (bare) quantities. We express Z_{ij} as

$$Z_{ij} = \delta_{ij} + \frac{g^2}{16\pi^2\epsilon} X_{ij} \quad (i, j = 1, \dots, n-2, m, E). \quad (14)$$

The relevant components of X_{ij} are given as follows:

$$\begin{aligned} X_{lj} &= C_G \frac{(j+1)(j+2)}{(l+1)(l+2)(l-j)} \\ &\quad + (2C_F - C_G) \left[(-1)^{l+j} \frac{n-2C_{j-1}}{n-2C_{l-1}} \frac{(n-1+l-j)}{(n-1)(l-j)} + \frac{2(-1)^j}{l(l+1)(l+2)} {}_lC_j \right] \\ &\quad (1 \leq j \leq l-1), \quad (15) \\ X_{ll} &= C_G \left(\frac{1}{l} - \frac{1}{l+1} - \frac{1}{l+2} - \frac{1}{n-l} - S_l - S_{n-l-1} \right) \end{aligned}$$

$$\begin{aligned}
& + (2C_F - C_G) \left[\frac{1}{n-1} + \frac{2(-1)^l}{l(l+1)(l+2)} - \frac{(-1)^l}{n-l} \right] \\
& + C_F (3 - 2S_l - 2S_{n-l-1}), \tag{16}
\end{aligned}$$

$$\begin{aligned}
X_{lj} &= C_G \frac{(n-1-j)(n-j)}{(n-1-l)(n-l)(j-l)} \\
& + (2C_F - C_G) \left[(-1)^{l+j} \frac{n-2}{n-2} \frac{C_j}{C_l} \frac{(n-1-l+j)}{(n-1)(j-l)} + (-1)^{n-j} \frac{n-2-l}{n-l} \frac{C_{n-2-j}}{C_{n-2-l}} \right] \\
& (l+1 \leq j \leq n-2), \tag{17}
\end{aligned}$$

$$X_{lm} = \frac{4C_F}{nl(l+1)(l+2)}, \quad X_{lE} = \frac{2C_F}{(n-1)(l+1)(l+2)}, \quad X_{mm} = -4C_F S_{n-1}. \tag{18}$$

For the physical quantity (moments), only the Z_{lj} , Z_{lm} , Z_{mm} components of the renormalization matrix give the contributions since the physical matrix element of the EOM operators turns out to be zero. With the above X_{ij} , the anomalous dimension matrix for the twist-3 operators becomes,

$$\gamma_{ij} = -\frac{g^2}{8\pi^2} X_{ij}. \tag{19}$$

This matrix enters into the renormalization group equation for the Wilson's coefficient function E_i associated with the corresponding operator as,

$$\left[\left(\mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} - \gamma_m m \frac{\partial}{\partial m} \right) \delta_{ij} - \gamma_{ij} \right] E_i = 0.$$

In the present study, we have obtained the anomalous dimension for the twist-3 operators which contribute to $g_2(x, Q^2)$. We performed the calculation with the manifest Lorentz covariance being kept. We adopted the Feynman gauge and dimensional regularization. We have chosen the operators which include the gluon field strength explicitly as an independent operator's basis although this choice of basis is never compulsory. To identify the renormalization constants correctly, the off-shell Green's functions are considered. We have shown that the EOM operators play an important role to complete the renormalization of composite operators and the structure of the renormalization constant matrix takes the triangular form expected from the general argument [9].

If we could calculate the “on-shell” matrix elements of composite operators in terms of purely perturbative Feynman graphs, we could obtain the enough informations without considering the EOM operators. However the infrared singularity coming from the collinear configuration can not be regulated [10]. Therefore we believe that calculating the off-shell Green’s functions is the safest method to obtain the anomalous dimensions.

As a technical issue, we have used the projection introduced in Ref.[8] to avoid the complexity stemmed from the fact that there are a lot of EOM operators which are not gauge invariant. This projection may have some relation to the case in which the light-cone gauge fixing $\Delta^\mu A_\mu = 0$ is adopted since this gauge does not discriminate between the covariant and the partial derivative in the composite operators.

Since the authors of Ref.[3] adopted the on-shell scheme to extract the anomalous dimensions, we can not compare our results with theirs graph by graph. However our final results are not in agreement with theirs although our definition of composite operators and choice of gauge are the same as those in Ref.[3]. If one replaces the index i by $n - 1 - i$ in their results, the both results become the same.

On the other hand, our final results agree with those of Ref.[4]. The approach adopted by them is quite different from ours. The main differences are : (a) they considered the physical quantities (structure functions) from the beginning, so the EOM operators do not appear explicitly, (b) they adopted the axial gauge to derive the GLAP-type evolution equation. In their analyses, the evolution of the quark bilinear operator (1) was also considered. Therefore it seems that their choice of the operator basis is different from ours at the intermediate stage of calculations. By including the quark bilinear operator (1), we can eliminate the gauge invariant EOM operator from the independent operator basis (see (5)). Furthermore, they included some contributions from the “one-particle-reducible” diagrams to take into account the circumstance that the partons can not be regarded as located on the mass shell [4]. This may cor-

respond to the fact that even if the gauge invariant EOM operator is discarded, the gauge non-invariant EOM operators turn out to mix through the renormalization [6].

We expect that future precise measurements on g_2 will clarify the effect of twist-3 operators which may be the first quantity to see the higher-twist effect in QCD.

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Figure Captions

Fig. 1 One-loop corrections to $\Delta \cdot R_{n,l}^\sigma$.

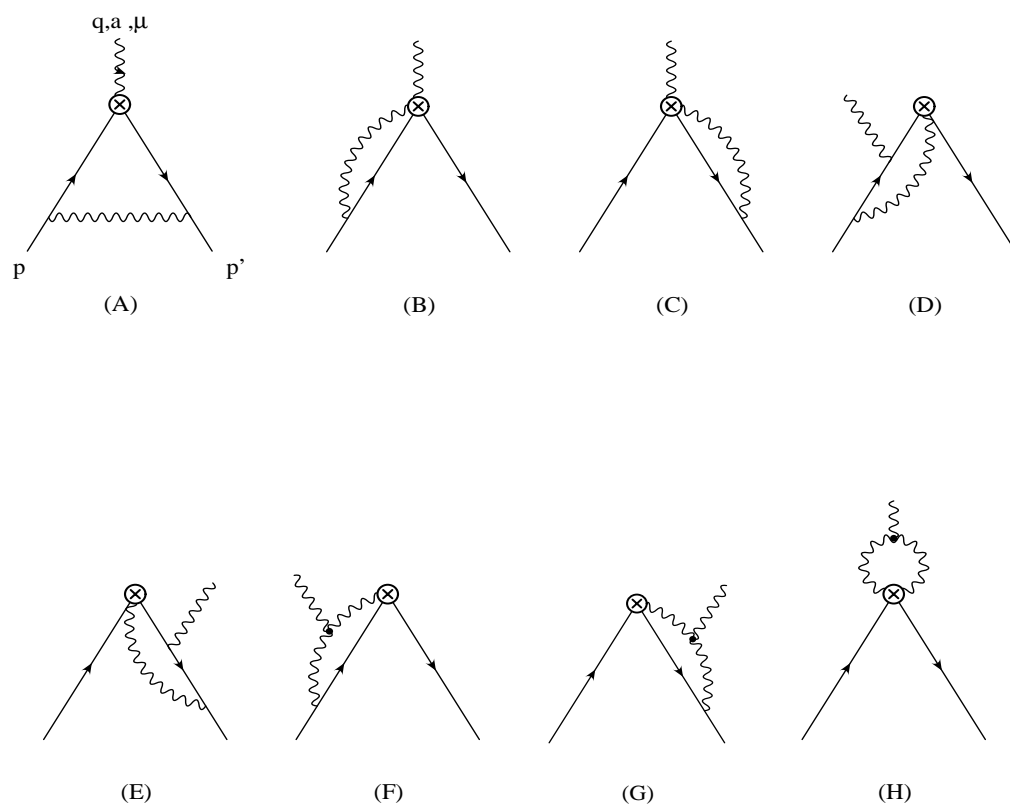


Fig.1